U.G. 2nd Semester Examination - 2022

BCA

[HONOURS]

Course Code : BBCAGEHT2

Course Title: Mathematics-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

a) Define odd function with examples.

b) Show that the series, $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$ is convergent.

c) Evaluate : $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$.

d) Find the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{2/3} = x^3 \frac{dy}{dx}$.

e) If $y = x^x$ then find $\frac{dy}{dx}$.

f) Evaluate: $\frac{d}{dx}\left(x^2 \log x\right)$.

g) $\int_{-1}^{1} xe^{x^2} dx = ?$

h) Differentiate sin x with respect to x^2.

i) Find the integrating factor of the differential equation. $\frac{dy}{dx} + \frac{2}{x} y = x^3$.

j) Show that, $\int_0^1 \frac{1-x}{1+x} dx = 2\log 2 - 1$.

k) Define homogeneous function of two variables.

l) If $f(x) = |x|$, then show that $f'(o)$ does not exist.

m) State Euler's theorem on Homogeneous function.

n) Define monotonic increasing sequence with example.

o) Find the differential equation from the relation $y = Ae^{2x} + Be^{3x}$, where A and B are constant.

2. Answer any **five** questions: 2×5=10

a) State Cauchy's general principle of convergence.

b) Find the sum of the following series

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+...$$

c) Is the function f(x), defined by

$$f(x) = \begin{cases} x^2, & \text{if } x \le 1 \\ x, & \text{if } x > 1 \end{cases}$$ continuous at x = 1.

d) Write down the formula for the integration of a product of two function.

e) Write down the geometrical meaning of Rolle's theorem.

f) Is the sequence 1, 2, 2^2, 2^3, ... convergent? Justify.

g) Evaluate : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}}dx$.

h) Evaluate: $\frac{1}{D^2+1}$ sin 2x.

3. Answer any **two** questions: 5×2=10

a) Prove that:

$$\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\frac{n+3}{n^2+3^2}+...+\frac{1}{n}\right]$$

$$=\frac{\pi}{4}+\frac{1}{2}\log 2.$$

b) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that,

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$$

c) Use Lagrange's theorem to prove that:

$1 + x < e^x < 1 + xe^x, \ \forall x > 0$

4. Answer any **one** question: 10×1=10

a) i) Find the differential equation of all circle having a constant radius.

ii) Evaluate: $\lim_{x\to\infty}(1+ax)^{\frac{1}{x}}$.

iii) State and prove Lagrange's Mean Value theorem. 3+2+5

b) i) Prove that the sequence $\left\{\frac{4n+3}{n+3}\right\}$ is monotone increasing and bounded.

ii) Test the convergence of the series

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+...\infty$$

5+5

c) i) Evaluate: $\int_0^1 \frac{dx}{\sqrt{2x - x^2}}$

ii) Solve: $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y(\log x)^2}{x^2}$

iii) Solve: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

3+3+4

———————