U.G. 1st Semester Examination - 2021

BCA

Course Code : BBCAGEHT1

Course Title : Mathematics-I

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: 1×10=10

a) If $A = \{x : x = \text{odd integer} \le 15\}$ and

$B = \{x : x = \text{prime number} < 15\}$

find $A \cap B$ and $A \cup B$

b) If the function $f : R \to R$ be defined by $f(x) = x^2 + 1$, then find $f^{-1}(17)$.

c) What are the generator of the cyclic group $\{1, -1, i, -i\}$ under usual multiplication.

d) If ω be an imaginary cube root of unity, then prove that

$(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8) = 9$.

e) Show that $x^4 + 2x^3 - 13x^2 - 14x + 24$ is exactly divisible by $(x+4)$.

f) If $\bar{a} = 3\hat{i} - 2\hat{j} + \hat{k}, \bar{b} = 2\hat{i} - \hat{j} - 3\hat{k}$ & $\bar{c} = \hat{i} - 2\hat{j} - 2\hat{k}$ then find $(2\bar{a} - 3\bar{b} + 4\bar{c})$.

g) Find the equation to the curve $3x^2 + 3y^2 + 6x - 18y = 14$ referred to parallel axes through the point $(-1, 3)$.

h) Find the transpose of the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$.

i) Find the modulus and amplitude of $(\sqrt{3} + i)$.

j) If $A = \{x : x \text{ is prime} < 15\}$ and $B = \{x : x \text{ is +ve integer} \le 15\}$ then find A–B.

k) Find the angle between the pair of straight lines $x^2 + 2hxy - y^2 = 0$.

l) Define orthogonal matrix.

m) Prove that the vectors $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{b} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are perpendicular to each other.

n) If R be a relation from a finite set A having 3 elements to a finite set B having 5 elements then find the number of relation from A to B.

o) If the function $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x)=2x+7$. Find $f^{-1}(7)$.

2. Answer any **five** questions: $2\times5=10$

a) If α,β,γ be the roots of the equation $x^3+px^2+qx+r=0$, then find $\Sigma\alpha^2$.

b) Without expanding the determinants prove that

$$\begin{vmatrix} a & d & 3a-4d \\ b & e & 3b-4e \\ e & f & 3e-4f \end{vmatrix} = 0$$

c) Examine if the set $S=\{(x,y,z)\in\mathbb{R}^3 : x+y+z=0\}$ is a subspace of $\mathbb{R}^3$.

d) Show that $\left[\bar{\alpha}+\bar{\beta}, \bar{\beta}+\bar{\gamma}, \bar{\gamma}+\bar{\alpha}\right]=2\left[\bar{\alpha}\bar{\beta}\bar{\gamma}\right]$

e) Find the nature of the roots of the equation $x^4+x^3+x^2+1=0$.

f) Find the general value of i^i.

g) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

h) If $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=3x$, $x\in\mathbb{R}$ and $G:\mathbb{R}\to\mathbb{R}$ be defined by $g(x)=\frac{x}{3}, x\in\mathbb{R}$. Show that $f_o g = g_o f$.

3. Answer any **two** questions: $5\times2=10$

a) If m and n be real, then show that

$$\text{Log}(m+in)=\tfrac{1}{2}\log(m^2+n^2)+i\left(2k\pi+\tan^{-1}\frac{n}{m}\right).$$

b) If α,β,γ be the roots of the equation $2x^3+x^2+x+1=0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha},\frac{1+\beta}{1-\beta},\frac{1+\gamma}{1-\gamma}$.

c) Let A, B, C be three subsets of a universal set S. Prove that $A-(B\cap C)=(A-B)\cup(A-C)$.

4. Answer any **one** question: $10\times1=10$

a) i) If ax+by transforms to $a'x'+b'y'$ under rotation of axes, then show that $a^2+b^2=a'^2+b'^2$.

ii) If $\bar{\alpha}=\hat{i}+\hat{j}-2\hat{k}$, and $\bar{\beta}=-\hat{i}+2\hat{j}+3\hat{k}$ and $\bar{\gamma}=5\hat{i}+8\hat{k}$, then determine scalars c and d such that $\bar{\gamma}-c\bar{x}-d\bar{\beta}$ is perpendicular to both $\bar{\alpha}$ and $\bar{\beta}$.

iii) Prove that the locus of the middle points of focal chords of a conic $\frac{l}{r}=1+e\cos\theta$ is $r^2(1-e^2\cos^2\alpha)=rle\cos\alpha$.

$3+3+4=10$

b) i) Find a basis and dimension of subspace of ω where $\omega = \{(x, y, z) \in \mathbb{R}^3 : x+2y-z=0,\ 2x-y+3z=0\}$

ii) Prove that the set of vectors $\{(1,2,2),(2,1,2),(2,2,1)\}$ is linearly independent in $\mathbb{R}^3$.

iii) On the curve $\frac{l}{r} = 1 - \cos\theta$, find the point with smallest radius vector.

5+3+2=10

c) i) Solve by Cardon's method of the equation $x^3 - 6x - 9 = 0$

ii) Solve the given equations by Cramer's rule

$$x + 2y + 3z = 6$$
$$2x + 4y + z = 7$$
$$3x + 2y + 9z = 14$$

iii) Find the value of $\sqrt[3]{i} + \sqrt[3]{-i}$, where $\sqrt[3]{z}$ is the principal cube root of z.

5+3+2=10
